Wigner Crystals

On the Theoretical Prediction and the First Imaging

Louis Böhm

USP IFSC

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1. Motivation

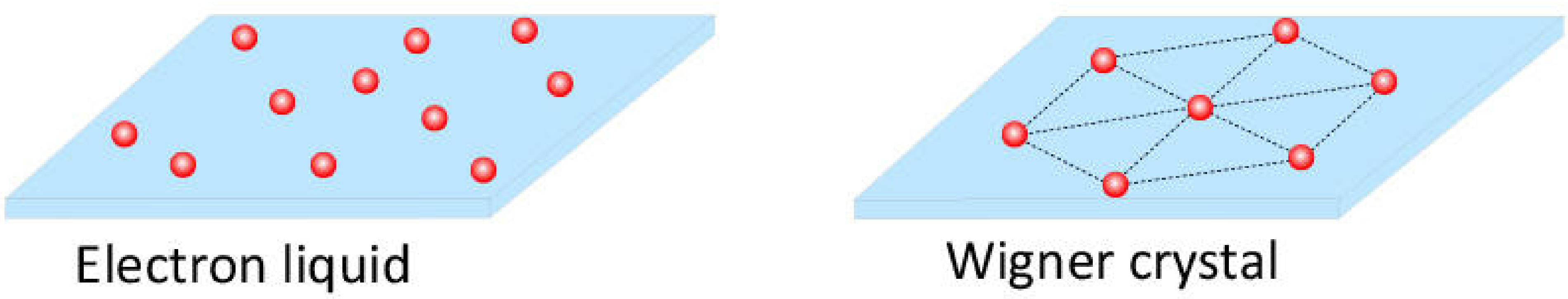


Figure 1: [1]

Definition 1.1 Wigner Crystals: A phase of electrons that occurs when the Coulomb potential between the electrons dominates over their kinetic energy. The electrons localize into a regular lattice structure to minimize their energy.

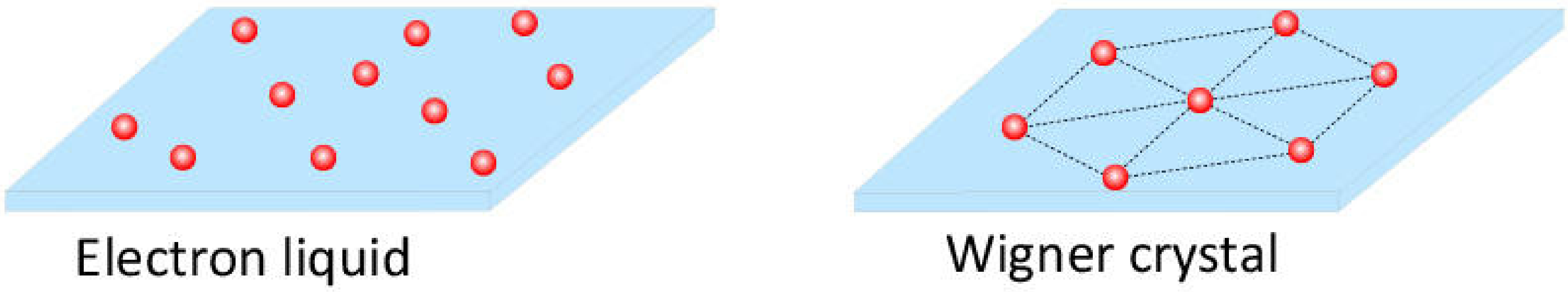
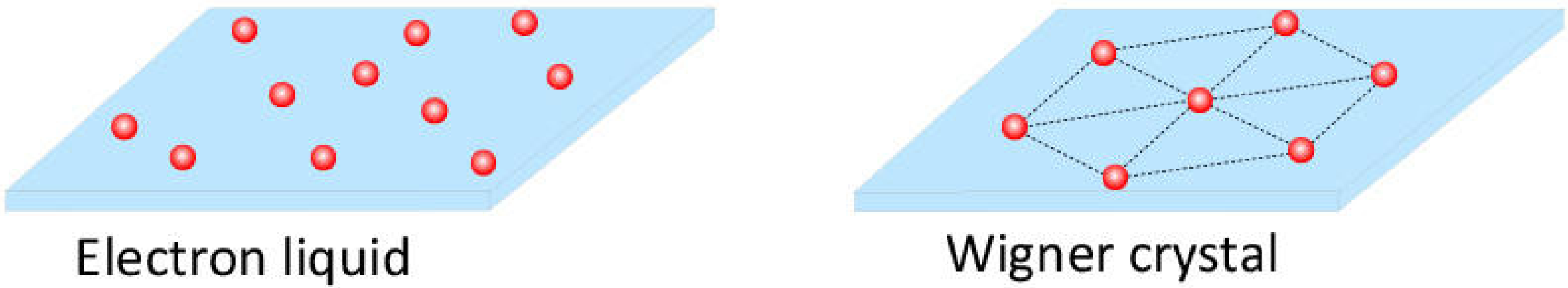


Figure 2: [1]

Historical context of Wigner crystals:

- 1934: Theoretical prediction by Eugene Wigner [2].



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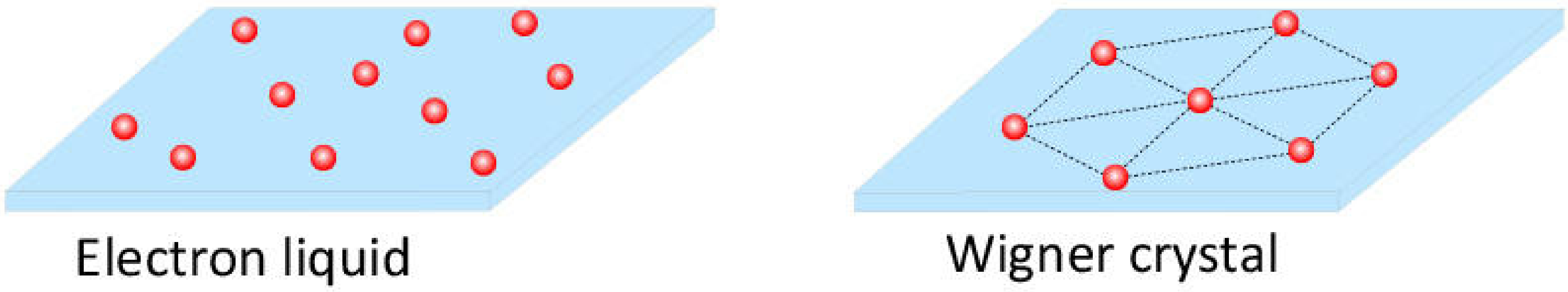
- 1979: First indirect observation by C. Grimes and G. Adams [3].



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Historical context of Wigner crystals:

- 1934: Theoretical prediction by Eugene Wigner [2].
- 1979: First indirect observation by C. Grimes and G. Adams [3].
- 2024: First imaging of Wigner crystals by Y. Tsui and colleagues [4].

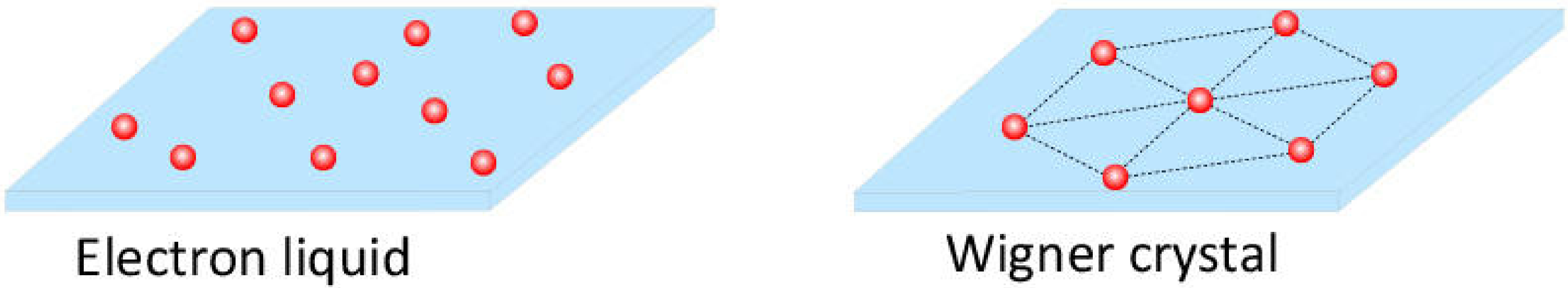
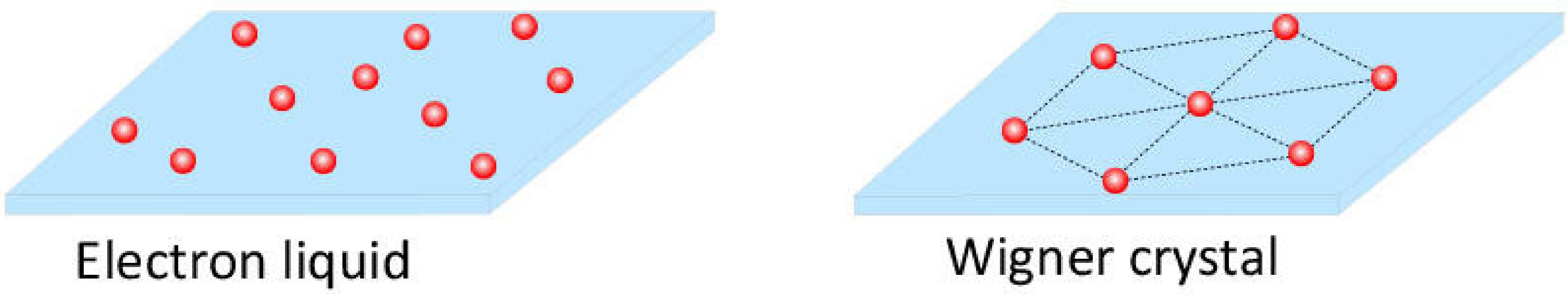


Figure 3: [1]

Research on Wigner crystals is important to better understand



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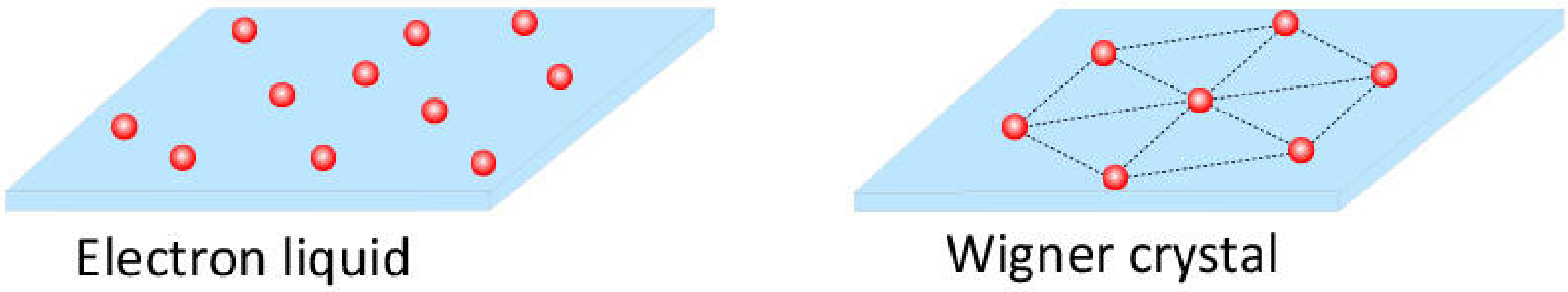
Research on Wigner crystals is important to better understand

- quantum phase transitions.



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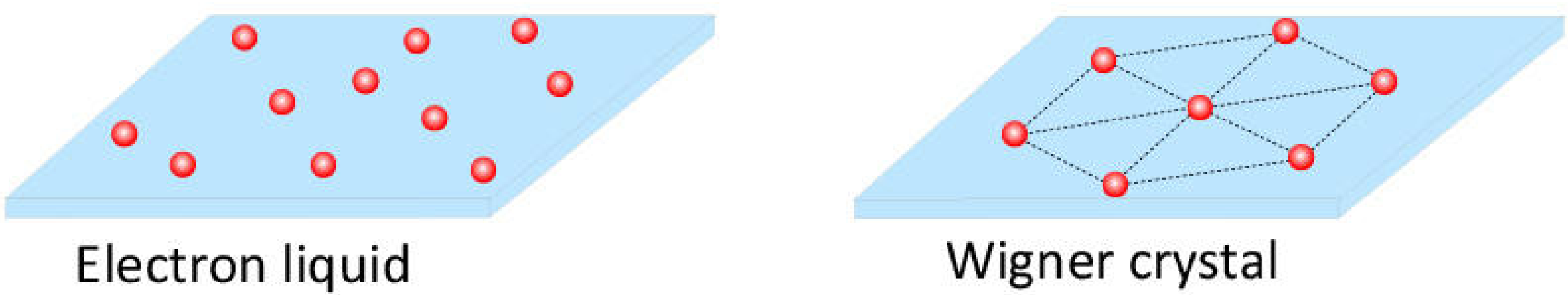

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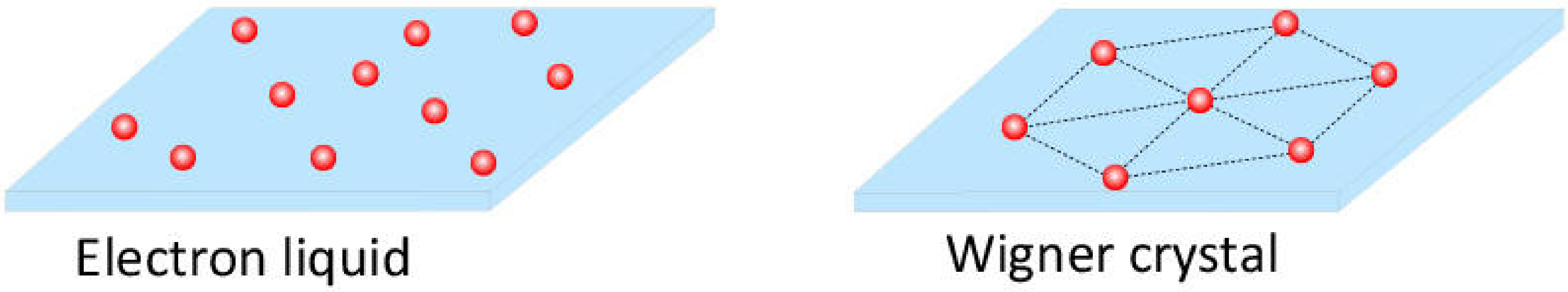
- transport phenomena in specific materials.



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→ Combines quantum mechanics and condensed matter physics.

Contents

2. Theory Background 1: Jellium Model

- Expands on the free electron model.

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- Most useful when valence electrons,
  - are de-localized.
  - do not participate in chemical bonding.

Important parameter:

Definition 2.2 Wigner-Seitz Radius r_s: Average distance between a pair of valence electrons in a uniform electron gas.

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In two dimensions:

$$r_s = \frac{1}{a_B \sqrt{\pi \mathrm{n}}}$$

3. Conditions for Wigner Crystallization

3.1 Energy Considerations

Conditions for Wigner crystallization: $U_C(r_s) > E_{\text{kin}}(r_s)$

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- Kinetic energy $E_{\text{kin}}(r_s)$ (2D):

$$E_{\text{kin}}(r_s) = \frac{\hbar^2}{m r_s^2 a_B^2} \propto \frac{1}{r_s^2}$$

3.2 Phase Boundary

Assume: $P = \text{const.}, T = \text{const.}$

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$$\rightarrow P = CT^4$$

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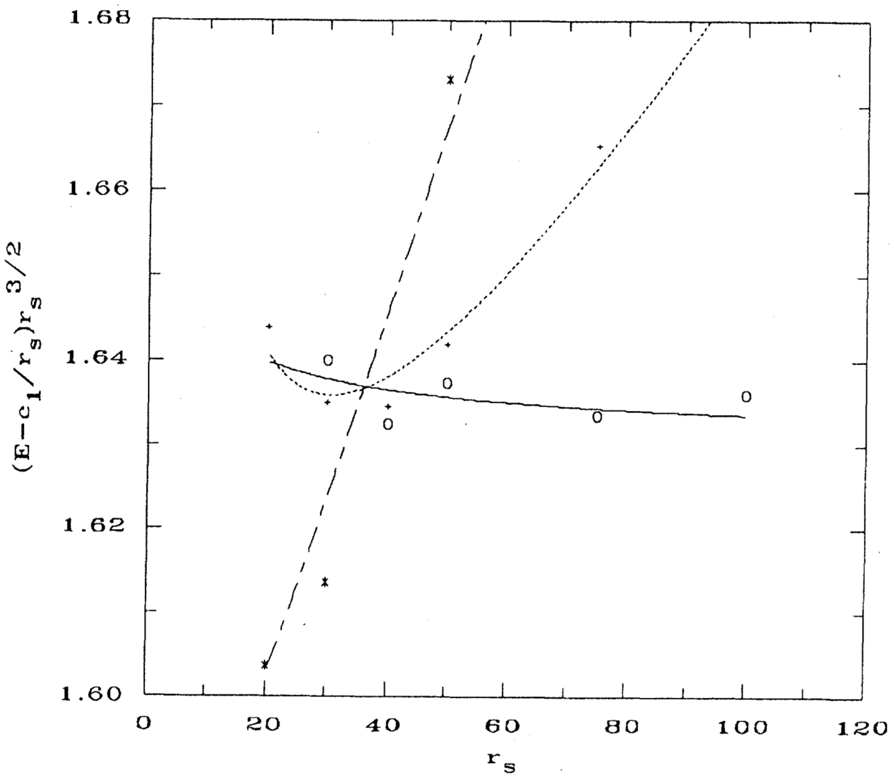
- Configurations R are positions of electrons.
- Trial wave function $\Psi_T(R)$ is an initial guess of the ground state.
- $\Psi_T(R)$ encapsulates the physics and symmetries of the system.

3.3 Computational Methods



o, solid: crystal

+, dotted: liquid

⋆, dashed: gas

T=0K

Figure 10: [5]

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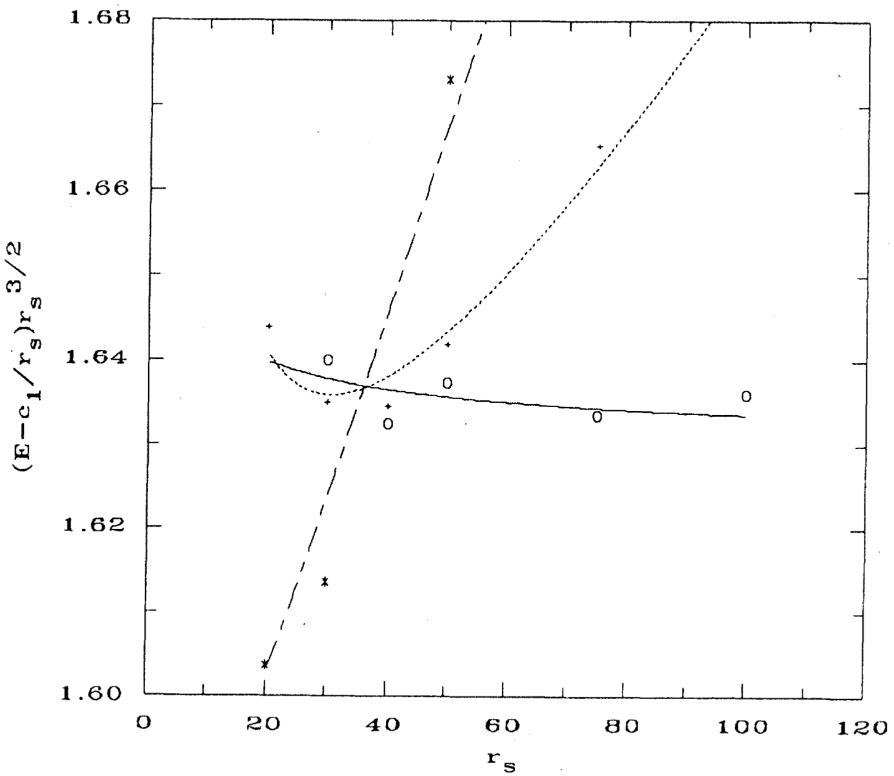

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Figure 10: [5]

Expected value of r_s to achieve Wigner crystallization: $r_s = 37 \pm 5$

4. Theory Background 2: Landau Levels

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$$E_n = \hbar\omega_c\left(n + \frac{1}{2}\right), \quad \text{with } \omega_c = \frac{|eB|}{m}$$

Landau radius r_L:

$$r_L = \frac{\hbar}{m\omega_c}$$

5. Experimental Observations

5.1 Experimental Setup

Measurement: Scanning Tunneling Microscope (STM)

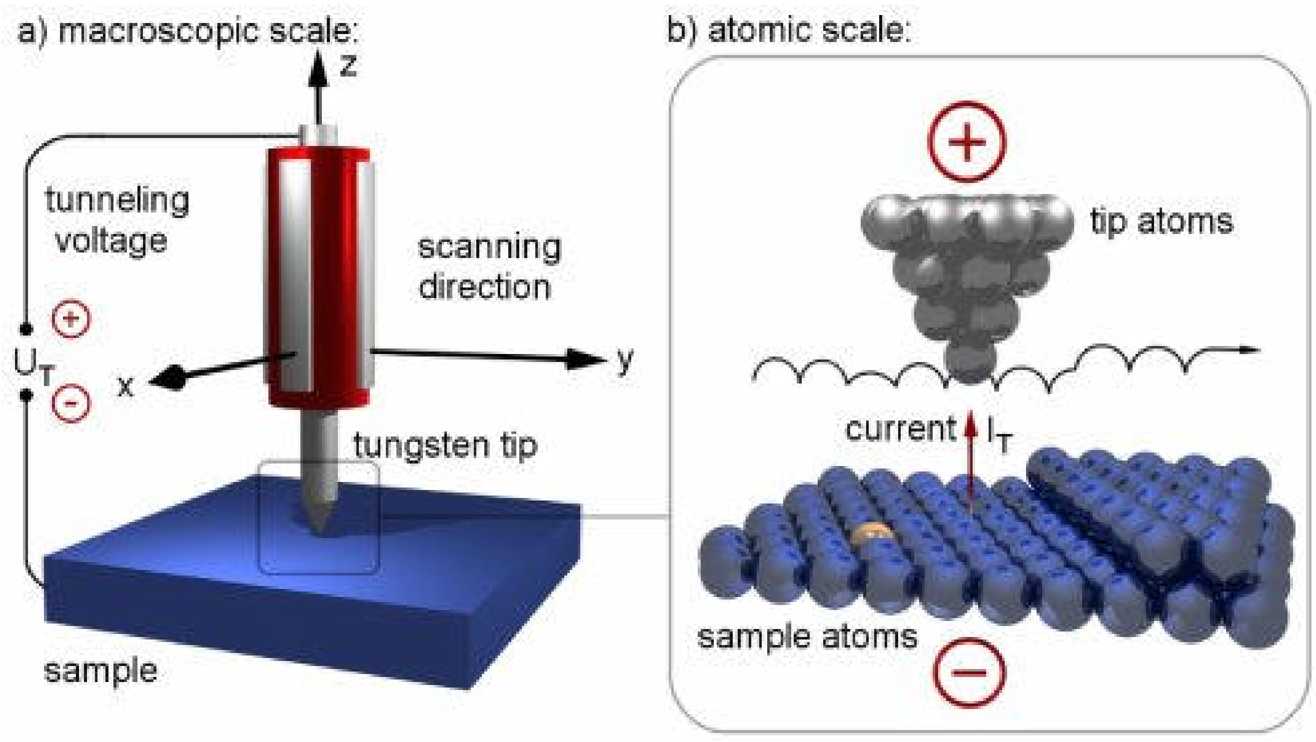

Figure 11: [6]

5.1 Experimental Setup

Sample: Ultra-pure bi-layer graphene

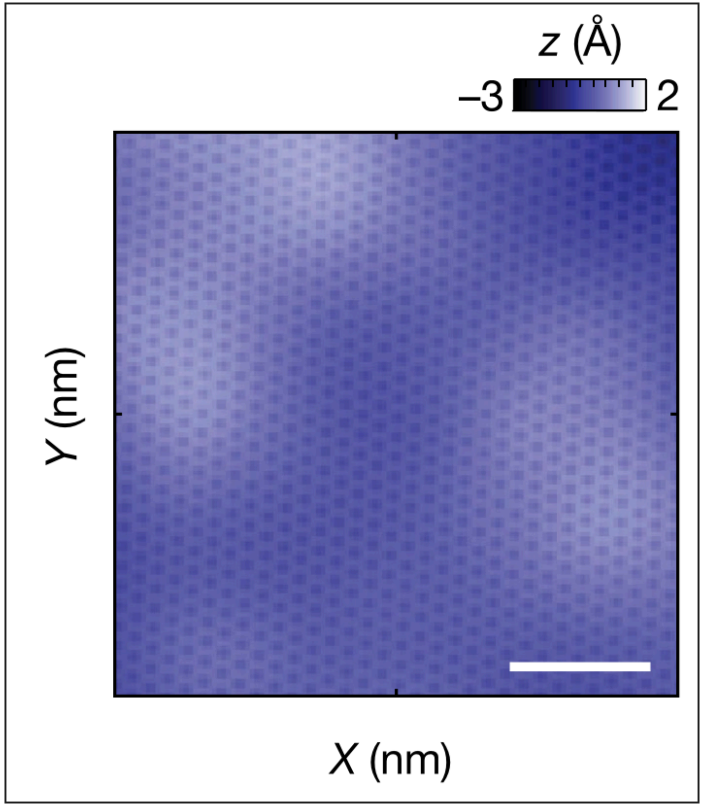

Figure 12: [4]

5.2 Results

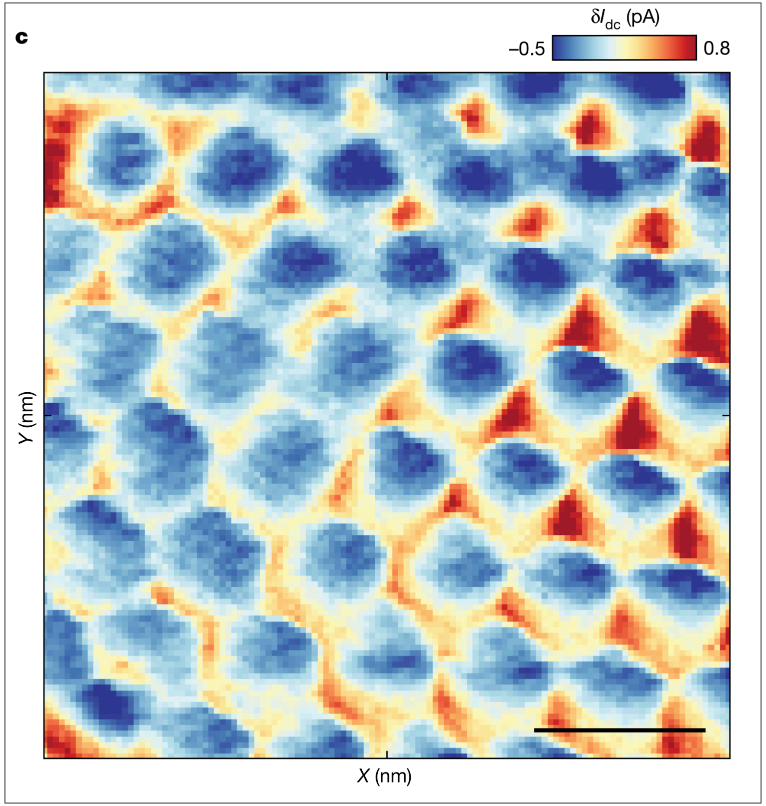

Figure 13: [4]

6. Conclusion and Outlook

- 90 years of advancements were needed to image Wigner crystals.

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- Still active research area.
- Many open questions remain, such as:
  - How does quantum phase transition occur in Wigner crystals?
  - Applications?
  - Imaging three-dimensional Wigner crystals?

[1] O. Morsch, “A crystal made of electrons.” [Online]. Available: https://ethz.ch/en/news-and-events/eth-news/news/2021/07/a-crystal-made-of-electrons.html

[2] E. Wigner, “On the Interaction of Electrons in Metals,” *Physical Review*, vol. 46, no. 11, pp. 1002–1011, 1934, doi: 10.1103/physrev.46.1002.

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[4] Y.-C. Tsui *et al.*, "Direct observation of a magnetic-field-induced Wigner crystal," *Nature*, vol. 628, no. 8007, pp. 287–292, 2024, doi: 10.1038/s41586-024-07212-7.

[5] B. Tanatar and D. M. Ceperley, "Ground state of the two-dimensional electron gas," *Physical Review B*, vol. 39, no. 8, pp. 5005–5016, 1989, doi: 10.1103/physrevb.39.5005.

[6] T. B. Group, "Scanning tunnel microscope." [Online]. Available: https://www.physics.rutgers.edu/Bartgroup/STM.htm